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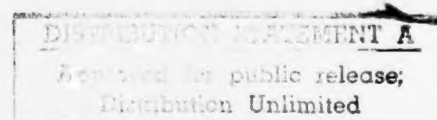
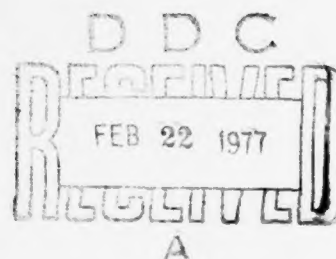
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METHODOLOGY FOR THE RETROSPECTIVE EVALUATION OF DECISION ANALYSIS

DECISIONS AND DESIGNS INCORPORATED

Judith Selvidge



ADVANCED DECISION TECHNOLOGY PROGRAM

CYBERNETICS TECHNOLOGY OFFICE
DEFENSE ADVANCED RESEARCH PROJECTS AGENCY
Office of Naval Research • Engineering Psychology Programs

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The objective of the Advanced Decision Technology Program is to develop and transfer to users in the Department of Defense advanced management technologies for decision making.

These technologies are based upon research in the areas of decision analysis, the behavioral sciences and interactive computer graphics.

The program is sponsored by the Cybernetics Technology Office of the Defense

Advanced Research Projects Agency and technical progress is monitored by the Office of Naval Research — Engineering Psychology Programs. Participants in the program are:

Decisions and Designs, Incorporated
The Oregon Research Institute
Perceptronics, Incorporated
Stanford University
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TECHNICAL REPORT 76-13

METHODOLOGY FOR THE RETROSPECTIVE EVALUATION OF DECISION ANALYSIS

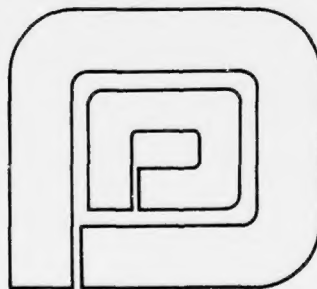
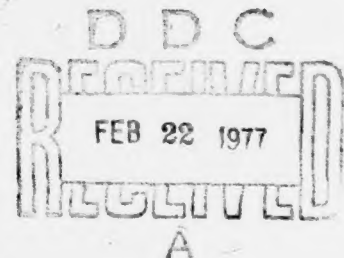
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Judith Selvidge

Sponsored by

Defense Advanced Research Projects Agency
Contract N00014-76-C-0074

September, 1976



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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) Methodology for the Retrospective Evaluation of Decision Analysis		5. TYPE OF REPORT & PERIOD COVERED Technical Sept., 75 - Sept. 76
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Judith Selvidge		8. CONTRACT OR GRANT NUMBER(s) N00014-76-C-0074
9. PERFORMING ORGANIZATION NAME AND ADDRESS Decisions and Designs, Incorporated Suite 600, 8400 Westpark Drive McLean, Virginia 22101		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS Defense Advanced Research Projects Agency (ARPA) 1400 Wilson Boulevard Arlington, Virginia 22217		12. REPORT DATE September, 1976
		13. NUMBER OF PAGES 40
14. MONITORING AGENCY NAME & ADDRESS (If different from Controlling Office) Office of Naval Research (Code 455) 800 North Quincy Street Arlington, Virginia 22217		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report) Approved for public release; distribution unlimited		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Evaluation of Decision Analysis Cost/Benefit of Decision Analysis Application of Decision Analysis Strengths and Weaknesses of Decision Analysis		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) Determining the effectiveness in practice of the formal techniques of decision analysis is difficult because typically the decision situations can not be replicated. This report presents an evaluation methodology in which the general degree of success of an analysis is determined, the specific benefits and costs, both direct and indirect, are enumerated and summarized, and a critique is made of the application of each of the steps in the decision analysis.		

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The purposes of this evaluation procedure are to encourage users of DA to review briefly and evaluate their experience after every analysis and to provide some general statistics about the strengths and weaknesses of decision analysis as it is practiced. Sample questionnaires are provided which can be used to elicit the desired information.

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SUMMARY

The formal techniques of decision analysis (DA) are a procedure for improving decision making under uncertainty. Although the logical foundations of decision analysis have been generally accepted, there is still some skepticism among potential users of the procedure about its effectiveness in practice. Several factors make it difficult to estimate the value of such an analysis even after it has been completed. Typically, because the problems studied by using DA involve particular decision situations which do not recur in the same form, an analyst cannot compare the decision recommended by the DA with that which would have been taken if no analysis were done or if some other type of analysis were undertaken. Since these comparisons, in fact, cannot be made, they must be guessed at by the analyst. Furthermore, because of the element of chance, the optimal decision taken under the DA (the decision option having the greatest expected utility) may not turn out to have the best outcome when the uncertainty is resolved.

Despite these difficulties, however, it is proposed in this paper that a systematic attempt be made to evaluate completed analyses. A methodology for this evaluation is presented. The first step of the evaluation is to determine the general degree of success of the analysis: Was the analysis completed and was its recommendation followed? Next, the specific benefits and costs, both direct and indirect, are enumerated and summarized. Finally, a critique is made of the application of each of the DA techniques.

The purposes of this evaluation procedure are, first, to encourage users of DA to review briefly and evaluate their experience after every analysis and, second, to provide some general statistics about the strengths and weaknesses of DA as it is practiced. To gather these general statistics, it is recommended that the evaluation procedure be carried out for 10 to 20 recent applications of DA by means of interviews with the analysts and users as well as by consulting the written reports of the analyses. This paper provides sample questionnaires which can be used to elicit the desired information.

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ACKNOWLEDGMENT

This research was supported by the Advanced Research Projects Agency of the Department of Defense and was monitored by the Office of Naval Research under Contract N00014-76-C-0074.

METHODOLOGY FOR THE RETROSPECTIVE EVALUATION OF DECISION ANALYSIS

1.0 DIFFICULTIES ENCOUNTERED IN THE RETROSPECTIVE EVALUATION OF DECISION ANALYSIS

The retrospective evaluation of an application of the techniques of decision analysis (DA) is difficult for a number of reasons. The formal procedures of DA cannot be followed exactly in most practical settings; an analysis, even if perfectly applied, may have no directly ascertainable benefits either because the recommended decision is the same as that which would have been taken without the analysis or because the uncertainty is ultimately resolved in a way which favors some other decision over that recommended by the analysis. Furthermore, the indirect advantages of a DA may be difficult to express in quantitative terms, as are some of the indirect costs of DA.

1.1 Methodology of Decision Analysis

Decision analysis is a method for selecting the best available option in decisions that involve some uncertainty.¹ The method is a formal procedure in which the decision problem is decomposed into several parts that can be treated separately. The results of these separate treatments are then aggregated to produce a recommended decision. The first step in the procedure is the enumeration of all decision options, relevant uncertain events and consequences resulting from combinations of decision options and event outcomes. Next, subjective probabilities are elicited for the different outcomes of each uncertain event, and the decision maker's utility for the various possible consequences is assessed. Finally, this information is combined by computing an expected utility value for each option. The option having the highest expected utility is recommended by the DA.

The theoretical foundations of DA have been the object of considerable debate in recent years. The inquiry centers on the use of subjective probabilities to express uncertainty,²

¹ See, for example, H. Raiffa, Decision Analysis (Reading, Massachusetts: Addison-Wesley, 1968); and R. V. Brown, A. S. Kahr, and C. R. Peterson, Decision Analysis for the Manager (New York: Holt, Rinehart, and Winston, 1974).

² L. J. Savage, The Foundations of Statistics (New York: Wiley, 1954).

multi-attributed utility functions as a measure of value,³ and maximization of expected utility as the criterion for rational decision making.⁴ At the same time, a great deal of experimentation has been carried out to test different techniques for actually eliciting the probability and utility values required by the DA procedure.⁵ On the bases of this theoretical and experimental work, many people believe that the use of DA ought to improve the decision making of an organization or an individual. It remains to be shown in a systematic way, however, that such improvements are, in fact, realized.

1.2 Practical Limitations of Applied Decision Analysis

Decision analysis as it is practiced does not conform perfectly to the theoretical model of DA. Differences between the actual and the ideal use of the method may limit the value of DA. For example, DA often simplifies real decision problems by omitting some of the options and uncertain events in order to speed up the analysis. Important factors to the decision problem may also be inadvertently left out. Obviously, the best decision option cannot be selected if it is not included in the analysis. The DA can also suffer if the numerical inputs, the probabilities measuring the uncertainty, and the utilities giving the value of the consequences are incorrect. In most cases, these figures will be approximations of the quantities being measured rather than exact numbers.

The possible lack of rigor when the DA methodology is applied is one limitation to the analysis; another limitation sometimes observed is the lack of acceptance of the recommendation of the DA simply because of resistance to the new method by the people concerned with the decision. Such resistance is often observed when new methods of management are introduced and is especially prevalent when the techniques are of a quantitative or mathematical nature.⁶

³R. Keeney and H. Raiffa, Decisions with Multiple Objectives (New York: Wiley, forthcoming).

⁴J. Von Neumann and O. Morgenstern, Theory of Games and Economic Behavior (Princeton, New Jersey: Princeton University Press, 1947).

⁵G. Huber, "Methods for Quantifying Subjective Probabilities and Multi-Attribute Utilities," Decision Sciences 5 (1974): 430-58.

⁶See, for example, C. J. Grayson, Jr., "Management Science and Business Practice," Harvard Business Review 51 (1973): 44.

1.3 Statistical Difficulties in Evaluating a Decision Analysis

When a DA is carefully, even flawlessly, performed, difficulties can still arise in the evaluation of the analysis. DA claims that, on the average and in the long run, the results of decision making by this technique should be superior to those obtained by some other method. A particular decision maker making a particular decision, on the other hand, may not be able to show this improvement and may not, in fact, be better off than with another technique.

First of all, the DA may simply recommend the decision that would have been taken anyway. In this case, the DA provides additional evidence suggesting that the selected decision is the optimal one, and this information may bring increased confidence or peace of mind to the decision maker. However, the value of such peace of mind could be less than the cost of the analysis. Sometimes, of course, the decision that would have been taken without this analysis (which we call the "non-DA decision") is not known. A decision analyst would maintain, that when the non-DA decision is known, its chances for being the same as that recommended by the DA depend, in part, on how good the decision maker is at intuitively integrating into the decision process all the factors which are handled explicitly by DA.

The other statistical difficulty arises from the qualifying phrase "on the average and in the long run." A particular decision may be correctly identified as the optimal one by the DA and yet after the fact turn out to be inferior to some other choice of option. One way in which this can come about is the occurrence of some unfavorable, unlikely outcome. This phenomenon is called in DA the "good decision, bad outcome" effect and is more generally recognized in the verse from Ecclesiastes:

. . . the race is not to the swift, nor the battle
to the strong, neither yet bread to the wise, nor
yet riches to men of understanding, nor yet favor to
men of skill, but time and chance happeneth to them
all.

The example of a simple lottery can be used to illustrate this effect. Suppose you are offered an opportunity to participate once in a lottery where a ball will be drawn from an urn containing 30 blue balls and 70 red balls. If a blue ball is drawn, you lose \$20, and if a red ball is drawn, you win \$100. This means that there is a 0.30 probability of losing \$20 and a 0.70 probability of winning \$100. This lottery has a positive expected value.⁷ Many

⁷ $(0.30 \times (-20)) + (0.70 \times \$100) = \$64.$

people, after taking into account their personal attitude toward risk, would find that the lottery also has a positive expected utility for them. The optimal decision for anyone for whom the lottery has a positive expected utility is to accept the gamble. If such a decision maker participates in the lottery, draws a blue ball, and loses \$20, we say that a bad (unfavorable) outcome has occurred. The initial decision to participate would still be considered by a decision analyst to be the correct choice.⁸ At the time the decision was made, the option to gamble seemed sensible; it is no reflection on the method used to arrive at a sensible decision that the possibility of losing later became a certainty in this particular instance.

The implication of this phenomenon for evaluating DA is that one can neither prove nor disprove the usefulness of DA by looking at the final outcome of one or two applications of the method. A large sample of similar applications must be considered to gather appropriate statistics. When an apparent "bad outcome" occurs, however, the analyst can try to get some idea of whether its occurrence results from vagaries of fortune or to errors in the analysis, such as incorrect assessments of the outcome probabilities or of the values of the consequences.

1.4 Indirect Benefits of Decision Analysis

The direct benefit of DA is intended to be, and theoretically is, an improvement in the decision selected. As described above, however, such direct benefit may not always be observed either because there is no change in the decision or because a bad outcome occurs. Whether or not this direct benefit results, however, there are a number of indirect ways in which the DA may be valuable.⁹ Two advantages of

⁸ This problem is discussed by R. S. Nickerson and C. E. Fechner, Decision Making and Training: A Review of Theoretical and Empirical Studies of Decision Making and Their Implications for the Training of Decision Makers, Technical Report NAVTRAEQUIPCEN 73-C-0128-1 (August 1975), pp. 160-164, who use the terms "logical soundness" to indicate a good decision and "effectiveness" to describe a decision having a good outcome.

⁹ In a study of U.S. governmental policy regarding the export of strategic materials to the Soviet Union, the principal benefits of the DA turned out to be indirect considerations such as the improvement in communication among decision makers. See S. R. Watson and R. V. Brown, Case Studies in the Value of Decision Analysis, Technical Report 75-10 (McLean, Virginia: Decisions and Designs, Incorporated, (October 1975), pp. 3-10 through 3-14 and A-1 through A-4.

the DA process are especially worth considering.

One indirect benefit of DA is the improvement in communication when, as is often the case for problems solved using DA, more than one person is a party to the decision. The process of DA makes explicit the important factors in the problem, namely, the options, the uncertainties, and the dimensions of value. This explicitness may reduce or help to resolve disagreements among the people concerned with the decision. Without the format of DA, two people might argue at length about which of two different decision options should be selected without realizing that their disagreement depended upon, for example, differences of opinion about the probabilities of the various outcomes of a relevant uncertain event. The DA format calls attention to these figures and if necessary, redirects the argument into the proper channels. In order to restrict disagreement to its essentials, the calculations of the DA can be repeated by using different estimates of probabilities. This step will show the true effect of different probability estimates upon the expected utility and the ultimate recommendation. The ability of DA to facilitate communication about a problem is useful not only for resolving differences within a group, but for communicating the justification of a decision to some oversight body as well.

Another indirect benefit of DA is that an individual involved in the analysis learns some of the skills needed to make correct numerical expressions of uncertainty and value. These numbers are an important input to the DA, and experience in their assessment obtained during one analysis should improve the precision of future analyses involving the same decision maker.

Unfortunately, these indirect benefits, such as improved communication and experience in the numerical assessment of probabilities and utilities, are rather intangible and, consequently, difficult to quantify.

1.5 Costs of Decision Analysis

The effectiveness of DA is a measure of its ability to improve decision making. The efficiency of DA is a measure of the economy with which this improvement is obtained. The efficiency is determined by comparing the costs of the analysis either with the benefits of the improvement or with

the costs of obtaining the improvement by some other method.¹⁰ The major costs are the cost of the analyst's time (the contract price if the DA is carried out by a consulting firm, as is often the case, at least for the first few analyses done by an organization) and the value of the time of people who work with the analyst to provide the substantive information important to the particular problem (mainly utility and probability assessments). Besides these direct costs of the analysis, there may be some indirect ones attending the use of DA if it requires the disruptions and discomfort of major departures from the normal decision-making procedures of the organization or individual.

Another possible large source of expense in the DA is the cost of making a bad decision. If the whole analysis is poorly done or if some important factor is inadvertently omitted, the computed expected utilities for different options may differ from their true expected utilities. This can lead to the selection by the decision maker of an option which is worse (in terms of true expected utility) than that which would have been chosen without the analysis. In such a case, the cost of being mistaken, which may be very substantial, should be included when the analysis is evaluated.

The principal costs incurred can be recorded during the analysis or recalled at the end of the analysis. Indirect costs, like indirect benefits, can be estimated at the end of the analysis, but they are difficult to quantify. The cost of having made a bad decision generally cannot be known until the important uncertainties are resolved, often some time after the analysis is completed.

1.6 Biases in Reporting the Data

Biases in the reporting of data needed for an evaluation also contribute to its difficulty. Two such biases are the

¹⁰ The possible difference in efficiency between DA and other decision procedures is shown in the example of a military decision which was analyzed both by DA and by means of a complex simulation model (carried out by a governmental laboratory). The recommended decision was the same for both approaches, but the DA cost only 15% as much as the simulation. See J. O. Chinnis; C. W. Kelly, III; R. D. Minckler; and M. F. O'Connor, Single Channel Ground and Airborne Radio System (SINCGARS) Evaluation Model, Technical Report DT/TR 75-2 (McLean, Virginia: Decisions and Designs, Incorporated, September 1975).

distortions due to hindsight and those due to a lack of candor. Studies of hindsight¹¹ have shown, for example, that after the fact people often report the particular outcome that occurred as having been inevitable, even though there was considerable prior uncertainty about the outcome. When people do acknowledge their prior uncertainty, the probabilities that they recall to express uncertainty are often biased in a direction that implies greater foresight than was actually the case. That is, the outcome that did occur is given high probabilities and those that did not, small probabilities. These biases can occur even when the subject is trying to provide accurate information. Another type of bias occurs when the evaluator consciously withholds or changes the data. An analyst working as a consultant, for example, may be reluctant to admit to having erred in an application of DA.

1.7 Evaluating Other Techniques

Many of these practical and statistical difficulties of evaluation, of course, are not unique to the evaluation of DA. They are also encountered in attempts to evaluate other decision-making procedures (such as simulation techniques) and formal models of management (in general systems and planning, programing, and budgeting procedures.) Managers and analysts, however, continue to use these techniques despite the lack of mathematically rigorous proof of the value of particular applications. Two possible reasons for the use of such techniques are the appeal of any apparently logical format of the techniques and the users' informal judgment that overall the benefits of the method outweigh its costs.

¹¹ See B. Fischhoff, "Hindsight is not Equal to Foresight," Oregon Research Institute Research Bulletin 14.13 (December 1974), for a discussion of the distortions of hindsight.

2.0 PROPOSED METHOD

2.1 Reasons for Attempting Evaluation

The method proposed here is designed for use in the retrospective evaluation of DA, despite the statistical difficulties described above and the biases arising from the subjective nature of the information recollected and reported. There are two main reasons for carrying out this evaluation. The first is to collect and summarize available data on the success or failure of DA for the information of potential users. These data can be summarized in a way to help support or refute some of the general claims and opinions about DA which form the "folklore" of decision theory. The other purpose of the evaluation is to provide a standard format for analysts who wish to review their work in a systematic way at the end of each specific analysis or study. This format is intended to help the analysts to determine the strengths and weaknesses of DA as they themselves practice it.

2.2 Possible Approaches

One way to evaluate an analysis is simply to write a paragraph or two discussing in a general way the degree to which the analysis seems to be worthwhile after the analysis is completed. Such qualitative summaries often appear at the end of articles describing an application of DA. Disadvantages of this method are the lack of uniformity in the information reported and a tendency to stress the favorable aspects of the analysis.

At the opposite end of the spectrum are extremely formal and quantitative approaches to evaluating DA which make use of the techniques of DA themselves. Matheson¹ and Rice² have presented mathematical methods for calculating the value of an analysis in a given problem whose structure has been determined, expressed in terms of the improvement in the decision maker's state of knowledge. Another method,

¹J. E. Matheson, "The Economic Value of Analysis and Computation," IEEE Journal on Systems, Science, and Cybernetics SSC-4 (1968): 325-32.

²T. R. Rice, "The Economics of Decision Making," Stanford University Research Report EES-SA-74-1 (1974): 1-121.

proposed by Watson and Brown³, applies the techniques of DA to evaluate a prospective decision analysis. In their method, a small decision analysis is carried out at several stages in the study of a problem, to compute the expected value of undertaking or continuing the main decision analysis. There are some possible drawbacks to these methods. For example, complex philosophical questions are raised when methods are used to evaluate themselves; the methods also require that some initial work be done before the study of interest to the user is attempted; and they stress direct benefits and costs of the analysis rather than including both direct and indirect factors.

Watson and Brown are testing the use of their method for computing the expected value of DA on actual problems. The method proposed in this paper for the retrospective evaluation of DA lies between the two extremes of a purely qualitative and a highly quantitative approach.

2.3 Characteristics of the Proposed Method

The procedure proposed in the present paper relies on anecdotal information about completed analyses; however, this information is to be collected in a thorough, systematic, and, to some extent, quantitative way. It is proposed that two general categories of information be collected: information permitting a macro evaluation of the steps of the procedure (strengths and weaknesses of different phases of the analysis), and information giving a general description of the problem and its setting.

The instruments for collecting the information are questionnaires to be filled in by the analyst and the user. Sample questionnaires (Exhibits 2-1 through 2-10) are supplied for collecting the data about a particular DA. Three summary sheets are also provided (Exhibits 2-11 through 2-13) on which important data from a number of studies can be consolidated. The questionnaires establish categories, call for monetary expressions of benefits and costs, and ask open-ended questions of the "if not, why not" and "give reasons" type. This format is in contrast to "Likert"⁴ type questionnaires

³S. R. Watson and R. V. Brown, Issues in the Value of Decision Analysis, Technical Report 75-9 (McLean, Virginia: Decisions and Designs, Incorporated, October 1975).

⁴R. Likert, "A Technique for the Measurement of Attitudes," Archives of Psychology No. 140 (1932): 44-53.

where the respondent is given a number of statements and for each is asked to express a degree of agreement by checking one of the following five responses: strongly disagree, disagree, uncertain, agree, and strongly agree. Nor does the proposed format purport to give detailed, exhaustive lists of possible response categories. For example, the degree-of-use questionnaire in Exhibit 2-2 provides five possible reasons for management's failure to follow the recommendations of a DA. In a large-scale study of the implementation of operations research and management science models, Sorenson and Zand⁵ designed a questionnaire in which sixty reasons are proposed for the failure to implement the results of a study. These latter types of questionnaires are not appropriate for the evaluation methodology presented here because of the small proposed sample-size (about 20 studies), the large number of variables of interest, and the exploratory nature of this evaluation methodology.

2.4 Macro Evaluation

A number of questions can be investigated or answered in the macro evaluation of a number of DA applications. These research questions are not stated in terms of hypotheses because of the present impossibility of collecting enough information to make rigorous statistical tests of such hypotheses.

2.4.1 Level of completion - In what percentage of the cases studied was the DA completed? For those not completed, at what stage and for what reason were they abandoned?

An analysis may be abandoned well before the expected value or utility of different decision options is computed. The reason may fall into one of three main categories. The first of these may tend to confirm the usefulness of the DA method, namely, the optimal decision option becomes obvious during the structuring of the problem (as the options are listed and the uncertainties and dimensions of value identified) or during the early stages of the probability assessment. Some people believe that, for many problems, the majority of the benefits of DA accrue during the early,

⁵ R. L. Sorensen and D. E. Zand, "Improving the Implementation of OR/MS Models by Applying the Lewin-Schein Theory of Change," Implementing Operations Research/Management Science, eds. R. L. Schultz and D. P. Slevin (New York: American Elsevier Publishing Co., 1975), p. 230.

structuring stages of the work. This view holds that the principal advantage of DA is its requirement that the important factors of the decision be expressed in an orderly manner, rather than its use of maximum expected utility as a decision factor. If many analyses fall into this category of abandonment, then some support will be given to that claim. The descriptive information (discussed in Section 2.6) will provide some indication of the type of problem or organization for which this is the case (e.g., first-time users or few dimensions of value).

The second category contains problems whose analysis is discontinued because the analysis is badly done (possibly because the problem was never suited for this kind of study). A third catch-all category of causes for curtailing a DA is "reasons unrelated to the DA" (e.g., the sponsoring group loses responsibility for the decision being studied.)

Exhibit 2-1 shows a questionnaire that can be used to collect this information. The responses to "if not, why not" are then sorted into the categories described above.

2.4.2 Degree of use - In what percentage of the cases studied was the recommended decision actually taken?

The crucial test of the value of a DA must be whether or not the recommendation of the analysis is followed. An interesting statistic, therefore, is the percentage of analyses in which the recommended decision option is adopted. This figure, like many others in the study, may be biased because successful DA's are more likely to be reported than unsuccessful ones. When the recommended decision is not taken, an attempt is made to identify reasons for the rejection of the results of the analysis. Some sample reasons are provided in Exhibit 2-2.

There is a tendency among the people who carry out DA to lose track of the problem studied as soon as the results of the analysis are delivered. This inquiry into the use of the results will help determine the truth of rumors that many careful and elaborate analyses are never implemented.

2.4.3 Comparisons among costs and benefits - How do the costs and expected benefits of the decision option recommended by the DA compare to the costs and expected benefits of the option selected by another method? How do expected costs compare with actual costs; estimated benefits with actual benefits?

LEVEL-OF-COMPLETION QUESTIONNAIRE

1. Was the DA completed?

Yes _____ No _____

If not, why not?

_____ Optimal decision option became obvious early
in the DA.

_____ Method turned out to be inappropriate for the
problem. Give reasons: _____

_____.

_____ User became impatient with the amount of time
and effort required by the DA.

_____ Resources (time, money, etc.) exhausted before
completion.

_____ Other. Specify: _____

_____.

EXHIBIT 2-1

DEGREE-OF-USE QUESTIONNAIRE

1. What was the option recommended by the analysis? _____

_____.

2. Was this decision taken?

Yes _____ No _____

If not, why not?

_____ Results of DA provided too late.

_____ DA incorrect (important factors omitted or
other errors). Specify: _____

_____.

_____ User did not understand the DA.

_____ "Political" factors intervened to prevent the
choice of the optimal decision.

_____ Other. Specify: _____

_____.

EXHIBIT 2-2

The most relevant comparison of the costs and benefits of the DA is with those costs and benefits which would have been incurred without the analysis (or with some other type of analysis). Unfortunately, these figures may be unavailable or very difficult to estimate.⁶ The first six columns of Exhibit 2-3 indicate the information needed to carry out a full computation of the net expected value of the analysis. The net expected value is defined as the total difference in expected benefit (between the DA and the other method) less the total difference in cost of the DA and the other method. The crucial information for purposes of comparison here is data about what the cost and expected benefits would have been if the DA had not been undertaken. In attempting to collect these data, the evaluator encounters one of several situations. These are:

- (1) Status quo case: If without the DA no action would have been taken, the expected costs and benefits of doing nothing can be taken as the base from which anything else will be measured and so set to zero. Thus, the "other method" columns under cost and benefit will contain only zero values. Then the expected benefit under the option recommended by the DA can be isolated and attributed entirely to the DA. The direct and indirect costs of the DA are accumulated during the analysis. The net expected of the analysis is simply the cost of the DA minus zero subtracted from the expected value of the decision option recommended by the DA minus zero.
- (2) Unaided decision known: The decision option that would have been taken without the DA is known for certain. The expected benefits computed by the DA for the two decision options can then be compared (assuming, as is generally the case, that the option selected by the unaided decision is also

⁶This problem of making reasonable deductions conditional on a false premise ("If the DA had not been done, what would have happened?") is discussed by logicians as the topic of "counterfactuals." See, for example, D. Lewis, Counterfactuals (Cambridge, Massachusetts: Harvard University Press, 1973), pp. 1-142.

COST-BENEFIT QUESTIONNAIRE

TYPE	COSTS OF THE ANALYSIS		EXPECTED BENEFIT FROM THE DECISION RECOMMENDED BY		ACTUAL BENEFIT FROM THE DECISION RECOMMENDED BY	
	DA METHOD	OTHER METHOD	DA METHOD	OTHER METHOD	DA METHOD	OTHER METHOD
DIRECT						
INDIRECT						

TOTAL

NET EXPECTED VALUE: Expected Benefit Difference - Cost Difference = _____

NET VALUE: Actual Benefit Difference - Cost Difference = _____

EXHIBIT 2-3

included as an option in the DA.)⁷ The costs of the DA are found as in situation (1). If the unaided decision option is known, then presumably the cost for arriving at that decision is known or can be easily estimated. All the information for columns one through six of Exhibit 2-3 is therefore available. (In the special case where the option selected by the unaided decision and that recommended by the DA are the same, the data in the benefits columns will be the same for "DA method" and "other method.") The differences in cost between the two methods can be calculated, as can the differences in expected benefit. The net expected value of the DA is the total difference in expected benefit less the total difference in costs.

- (3) Several possible unaided decision options: In situations where the unaided decision option is not known for certain, it may be that the evaluator can obtain probability assessments of the likelihood that each of the various different candidate options would have been chosen without the analysis. The "other method" columns in Exhibit 2-3 (columns 2 and 5) can then be filled with costs and expected benefits which are weighted averages of the costs and expected benefits for each of the candidate decision options. (The weights are the probabilities that each option would have been chosen.) Usually the decision option selected by the DA will be included as one of the candidate options for the unaided method; however, in some instances this decision option may be one generated by the DA and which would not have been considered otherwise. The expected benefit for each candidate option can be obtained from the DA since these candidates would normally all be included in the DA. On the other hand, the costs of arriving at each of these decisions by some method other than DA may be difficult to estimate.

⁷In a study of possible programs for the commercialization of synthetic fuels in which the Decision Analysis group at Stanford Research Institute carried out a DA, the difference in expected benefit between the expected unaided decision and that recommended by the DA was approximately \$3.8 billion. See Synthetic Fuels Commercialization Program, Vol. II, Cost/Benefit Analysis of Alternative Production Levels, Report Submitted by Synfuels Interagency Task Force to the President's Energy Resources Council (November 1975).

- (4) Unaided decision unknown: In some cases the analyst or evaluator may have no idea about what option would have been taken without the DA or even what alternatives would have been considered. In this case the net expected value of the DA would be estimated by considering only the costs of the DA in comparison to the expected benefits of its recommended decision option (column 4 minus column 1 in Exhibit 2-3.) This limited assessment of the expected value of the DA probably overestimates the value since the calculation reduces to that of situation (1) where it is assumed that the alternative method to the DA has no cost and yields no benefit. (If the unknown expected benefit of the other method is greater than its unknown cost, the net expected value of the DA will be too high.)

The net expected value of the DA is a measure that compares how well the analyst expects to do by carrying out a DA versus using some other method. As discussed earlier, however, the expected gains of an analysis may not be realized if, for example, the uncertainty is resolved in an unfavorable manner (if, in other words, a "bad outcome" occurs.) A second interesting comparison, therefore, is between the expected benefit under the decision option recommended by the DA and the actual benefit if that decision is taken. Differences between actual and expected benefits can also occur if some part of the analysis is incorrectly done.

As part of the evaluation, data are collected to show how often the DA changes the decision to be taken, how often the actual and expected benefits of the recommended decision differ, what the reasons are for the difference, and how much they differ. The last three columns of the questionnaire in Exhibit 2-4 provide a means for recording the actual benefits. Since only one option is taken, the figures will be the actual observed benefits in one column but for the other column will be estimates of what would have been observed had the other decision option been taken.

Exhibit 2-5 shows a questionnaire designed for collecting information about how often and why the expected and actual benefits differ. The data about indirect costs and benefits must be collected and reduced to monetary terms for use in determining the net value shown in Exhibit 2-3. This part of the evaluation may be quite difficult for some users or in some applications. The questionnaires shown in Exhibits 2-6 and 2-7 give a possible breakdown of the desired information but do not solve the problem of how the transformation to monetary terms is to be accomplished.

AIDED VERSUS UNAIDED OPTION QUESTIONNAIRE

1. What is the option recommended by the DA? _____
_____.

2. What option would have been taken without the DA? Was it
_____ Known? (Specify) _____
_____.

_____ Uncertain? (Give possibilities and their probabilities of having been selected)

Option

Probability

_____	_____
_____	_____
_____	_____
_____	_____

_____ Unknown?

3. If known, is it the same or different from that recommended
by the analysis? _____

EXHIBIT 2-4

ACTUAL AND EXPECTED BENEFITS QUESTIONNAIRE

1. Was the recommendation of the DA followed?
_____ Yes _____ No (if no, the "actual" benefits
are estimates)
2. Did actual benefits of the DA differ from the expected
benefits?
_____ Yes _____ No
3. If yes, how much was the difference (as a percentage of
the total benefit)? _____
4. Was the difference
_____ Favorable (greater benefits realized) or
_____ Unfavorable (few actual benefits)
5. What are the reasons for the difference?
_____ "Bad outcome" (give the probabilities assessed for
the desired outcome) _____
the outcome which occurred _____)
_____ Incorrect analysis (list the mistakes in the
analysis)

EXHIBIT 2-5

INDIRECT COSTS OF THE DECISION ANALYSIS

Check the types of costs incurred in the analysis. Estimate the amount of these costs in any natural units and then estimate the corresponding monetary value.

<u>Cost Type</u>	<u>Amount</u>	<u>Monetary Equivalent</u>
1. Time of employees working with the analyst	_____ days	= \$ _____
2. Others (specify)	_____	
3.	_____	
4.	_____	

EXHIBIT 2-6

INDIRECT BENEFITS OF THE DECISION ANALYSIS

Check the types of benefits obtained and estimate their value in monetary terms.

<u>Benefit Type</u>	<u>Monetary Equivalent</u>
1. Improved communication among group decision makers.	
2. Ease of obtaining approval of the decision by higher levels in the organization.	
3. Same model can be applied with minor changes to other problems.	
4. Practice in assessing probabilities and utilities.	
5. Other (specify)	

EXHIBIT 2-7

2.5 Micro Evaluation

During the micro evaluation of a DA, each of the main steps in the DA is examined in turn to see how well that part of the analysis was performed. The steps considered are: structuring the problem, assessing the probabilities and utilities, computing the expected utility of each option to find the one with the highest value, and testing the sensitivity of the recommendation of the DA to changes in structure and inputs. First, the analyst is asked to recall which steps seemed most helpful during the analysis by revealing important information which had not been considered before the DA was undertaken. Next, the analyst evaluates these steps in light of all that is known by the end of the analysis and after some of the uncertainty has been resolved. The questions to be answered at this point are: what important factors were completely overlooked and what inputs (probabilities or utilities) were overestimated or underestimated. Exhibit 2-8 presents a format in which these data can be collected.

For each example of a DA, these data provide information about which steps in the analysis were most helpful. This determination may depend strongly on the particular kind of problem analyzed (the relevant categories of problem have yet to be specified) or on the amount of experience of the analyst. The user of the recommendation of the DA may also be asked which aspects of the analysis seemed to be the most difficult, the easiest, the most helpful, and so on.

2.6 Problem Description

The purpose of the problem description section of the evaluation is to collect information about variables in the problem that may turn out to be correlated with the overall success of the DA or with the detailed evaluation of the different steps in the DA. These descriptive data deal with the magnitude of the problem, the amount of resources at risk, the analyst's and the user's level of experience with DA, and the type of organization making the decision. Exhibit 2-9 provides a format for collecting this information.

2.7 Summary Sheets

After a number of DA's have been evaluated by using the questionnaires shown in Exhibits 2-1 through 2-9, the results of this sample survey can be summarized in the format presented in Exhibits 2-10 through 2-13.

MICRO EVALUATION QUESTIONNAIRE

DURING THE ANALYSIS:

Note the contribution of each step in the analysis to the description and resolution of the problem during the analysis. Next rank these contributions in order of their overall importance to the DA. (These data give an indication of how much better DA is than a traditional method of problem solving).

AFTER THE ANALYSIS:

Note for each step in the analysis what important factors were omitted and what inputs were incorrectly assessed. (These data indicate how the DA as performed compares to a perfect DA).

STRUCTURE	Important elements revealed during the DA (list them)	Any important elements omitted? (list them, with reason omitted).																																																					
Generating options																																																							
Identifying Probabilities																																																							
Identifying dimensions of values																																																							
ASSESSMENTS	<table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <th colspan="3">Were Assessments</th><th rowspan="2">Results Surprising? Why?</th><th colspan="2">Were Assessments Incorrect?</th><th colspan="3">If yes, were they Under or Over Estimated?</th><th rowspan="2">Reasons</th></tr> <tr> <th>Hard?</th><th>Easy?</th><th></th><th>No</th><th>Yes</th><th>Estimated</th><th>Estimated</th></tr> <tr> <td>Probabilities</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr> <tr> <td>Incremental costs and Benefits</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr> <tr> <td>Weighting factors for different dimensions of value</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr> <tr> <td>Conversion to utilities</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr> </table>	Were Assessments			Results Surprising? Why?	Were Assessments Incorrect?		If yes, were they Under or Over Estimated?			Reasons	Hard?	Easy?		No	Yes	Estimated	Estimated	Probabilities									Incremental costs and Benefits									Weighting factors for different dimensions of value									Conversion to utilities									
Were Assessments			Results Surprising? Why?	Were Assessments Incorrect?		If yes, were they Under or Over Estimated?			Reasons																																														
Hard?	Easy?			No	Yes	Estimated	Estimated																																																
Probabilities																																																							
Incremental costs and Benefits																																																							
Weighting factors for different dimensions of value																																																							
Conversion to utilities																																																							
SENSITIVITY	Which assumptions or inputs were shown by the sensitivity analysis to be important? (specify)	Which critical assumptions or inputs did the sensitivity fail to detect and why?																																																					
Probabilities																																																							
Incremental costs and Benefits																																																							
Weighting factors for different dimensions of value																																																							
Conversion to utilities																																																							

EXHIBIT 2-8

DESCRIPTION OF THE DECISION PROBLEM

Resources at risk:

1. What is the amount (\$) at risk in this problem?
\$ _____ (or range)
2. What percentage is this of the annual budget of the
organizational unit making the decision?
_____ % (or range)

Magnitude and complexity of the problem:

3. a. Number of decision nodes _____.
b. Total number of options (summed over all
decision nodes) _____.
4. a. Number of uncertain events considered _____.
b. Total number of different event outcomes _____.
5. a. Number of dimensions of value _____.
b. Total number of outcomes evaluated (summed over
all different dimensions) _____.

Experience of analyst and user:

6. How much experience has the analyst had with this
type of problem? _____.
7. How many DA's has the user participated in? _____.
8. How much did the user participate in this DA?
_____.

Type of organization in which the decision will be made.

9. Profit _____, non-profit _____, governmental
(give agency _____).
10. Annual sales of organization _____.
11. Management style _____ centralized
_____ decentralized.

EXHIBIT 2-9

SUMMARY OF MACRO EVALUATION STATISTICS OVER THE TOTAL SAMPLE

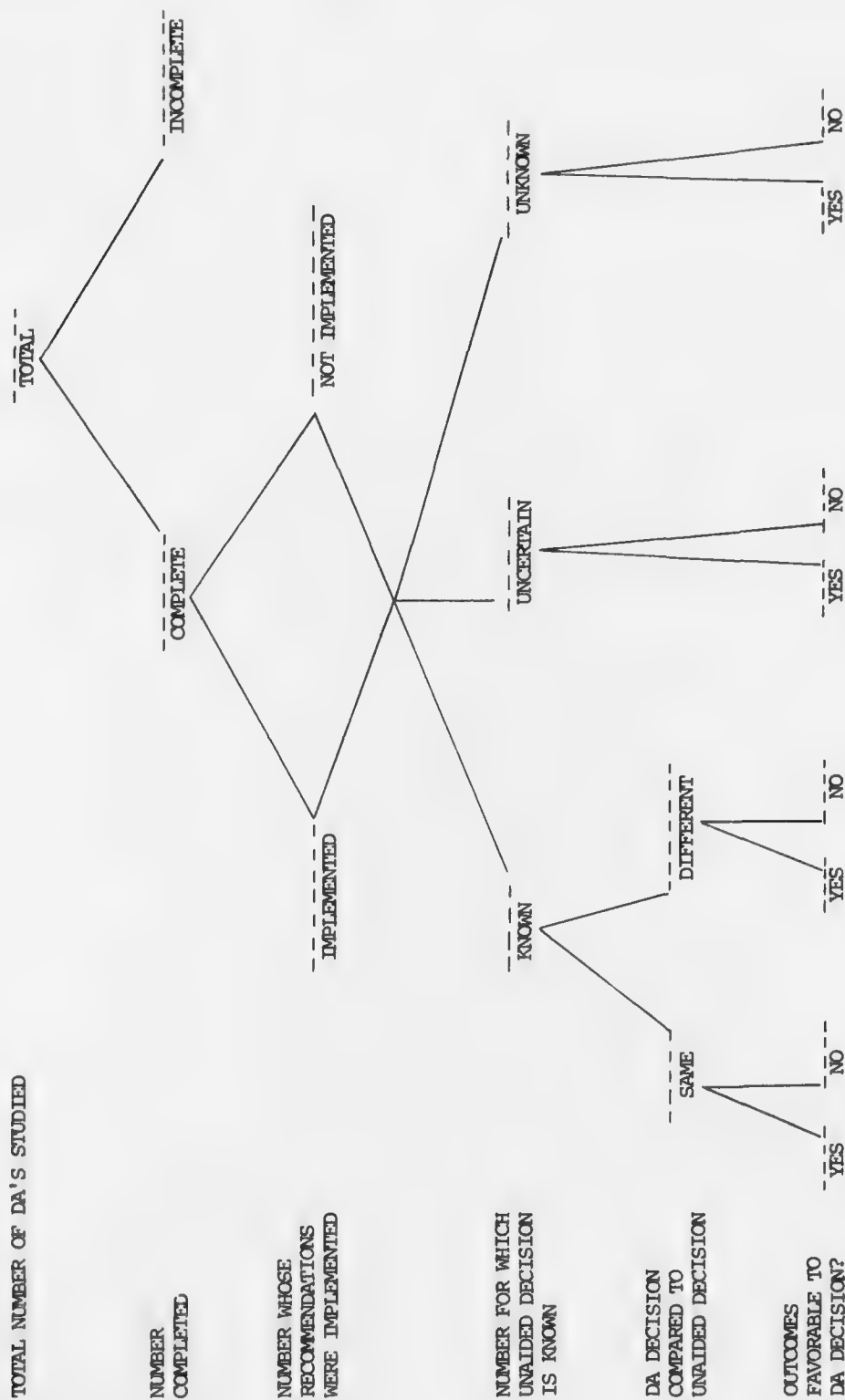


EXHIBIT 2-10

SUMMARY OF NET EXPECTED VALUE OF THE DA'S			
<u>DA APPLICATION</u>	<u>NET EXPECTED VALUE</u>	<u>NET ACTUAL VALUE</u>	<u>DIFFERENCE</u>
1.			
2.			
3.			
4.			
5.			
6.			
7.			
8.			
9.			
10.			
11.			
12.			
13.			
14.			
15.			
16.			
17.			
18.			
19.			
20.			
Average			

EXHIBIT 2-11

**SUMMARY OF THE MICRO EVALUATION
STATISTICS OVER THE WHOLE SAMPLE**

	DURING ANALYSIS, PERCENTAGE OF TIME EACH STEP WAS IDENTIFIED AS			AFTER THE ANALYSIS, PERCENTAGE OF TIME FACTORS WERE INCORRECT OR OMITTED	
STRUCTURE	Revealing Important New Elements			Elements Omitted	
Generating Options					
Identifying Probabilities					
Identifying Dimensions of Value					
ASSESSMENTS	Hard	Easy	Surprising	Correct	Incorrect
Probabilities					
Incremental Costs and Benefits					
Weighting Factors for Different Dimensions of Value					
Conversion to Utilities					
SENSITIVITY	Important to the Sensitivity			Critical Factors Undetected	
Probabilities					
Incremental Costs and Benefits					
Weighting Factors for Different Dimensions of Value					
Conversion to Utilities					

In each column, circle the largest percentage . Sample Size _____

EXHIBIT 2-12

SUMMARY OF THE DESCRIPTIVE INFORMATION
ABOUT PROBLEM SIZE

<u>APPLICATION</u>	<u>AMOUNT AT RISK</u>		<u>NUMBER OF</u>		
	<u>DOLLARS</u>	<u>PERCENTAGE OF ANNUAL SALES</u>	<u>DECISION NODES</u>	<u>EVENT NODES</u>	<u>VALUE DIMENSIONS</u>
1.					
2.					
3.					
4.					
5.					
6.					
7.					
8.					
9.					
10.					
11.					
12.					
13.					
14.					
15.					
16.					
17.					
18.					
19.					
20.					
Average:					

EXHIBIT 2-13

3.0 DISCUSSION

The purposes of this evaluation procedure are to establish a record of the long-run gains resulting from the use of DA (in general, or by a particular analyst or organization) and to encourage users and analysts to review their work on a regular basis at the end of each analysis.

It may be that the procedure recommended here is too much trouble, requiring, as it does, that the analyst and user spend time thinking about a problem after the DA has supposedly solved it. People's tendency at the end of a successful quantitative study is briefly to express pleasure with the outcome and then look for another possible application of the method. If, on the other hand, the study has turned out badly, people tend to put the unfavorable experience out of their minds. For a methodology of evaluation to overcome these tendencies, the evaluator must believe that the information demanded can be produced, that it is valuable, and that the sample questionnaires are an efficient way of collecting the information. These two points are presently being investigated by applying parts of the methodology to a number of past DA studies.

A more serious problem is the overall validity of the data. Can the indirect costs and benefits be computed in any reasonable way? Are the judgments about what would have happened without the application of DA nonsense because of problems of hindsight and lack of candor? Full-scale application of the evaluation methodology is proposed to help answer these questions. If distortions because of hindsight seem to be a serious problem, they can be lessened, for example, by having the users of the DA record formally before the analysis begins their initial impressions about such things as the probabilities and the decision which would be chosen without benefit of the DA. Frankness in assessing the factors used to determine the retrospective value of an analysis can perhaps be encouraged by reporting the results in an anonymous fashion.

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